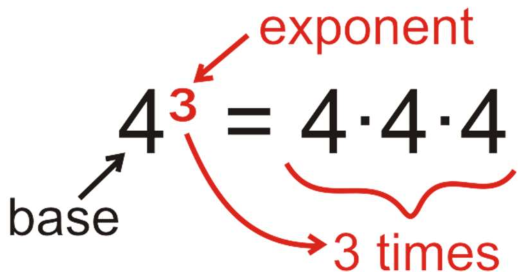
Exponents

An **exponent** (ex-po-nent) refers to the number of times a number is multiplied by itself.



Examples:

$5^2 = 5 \cdot 5 = 25$

$2^3 = 2 \cdot 2 \cdot 2 = 8$

$10^5 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000$

$1^7 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$

Tip #1: 10^n is a 1 with n zeros after it. (10^2=100; 10^3=1,000; 10^7=10,000,000)

Tip #2: 1^n is a 1 no matter what value is used for n.

<u>Vocabulary notes:</u>

- 5^2 can be read as 5 to the second power or 5 **squared.**
- 2^3 can be read as 2 to the third power or 2 **cubed.**

Higher exponents are read as the 4th power, etc.

Note: Any number raised to the zero power is 1.

$5^0 = 1$ and $342^0 = 1$, etc.

100,000	10,000	1,000	100	10	1	0.1	0.01	0.001	0.0001
100,000	10,000	1,000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$	$\frac{1}{10,000}$
10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten-thousandths
						 dime	 cent	 mill	

Note: Any number raised to the zero power is 1.

$5^0 = 1$ and $342^0 = 1$, etc.

Expanded Notation uses fractions for place value.

$$34.762 = 3\text{x}10 + 4\text{x}1 + 7\text{x}\frac{1}{10} + 6\text{x}\frac{1}{100} + 2\text{x}\frac{1}{1,000}$$

Exponential Notation uses exponents for place value.

$$34.762 = 3\text{x}10^1 + 4\text{x}10^0 + 7\text{x}\frac{1}{10^1} + 6\text{x}\frac{1}{10^2} + 2\text{x}\frac{1}{10^3}$$

Exponents and Integers

Be sure to watch the parentheses! Is the negative being raised to a power or is the number being raised and then you need the opposite?

Examples:

$5^2 = (5)\cdot(5) = 25$

$(-2)^4 = (-2)\cdot(-2)\cdot(-2)\cdot(-2) = +16$

$-(2)^4 = -(2)\cdot(2)\cdot(2)\cdot(2) = -16$

$-2^4 = -(2)\cdot(2)\cdot(2)\cdot(2) = -16$

Multiplication and Division with Exponents

Rules:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

Examples:

$$\frac{3^6}{3^2} = \frac{3\times3\times3\times3\times3\times3}{3\times3}$$
$$= 3^{(6-2)} = 3^4$$

$$10^5 \cdot 10^{18} = 10^{5+18}$$
$$= 10^{23}$$

Raising an Exponent to a Power

Rules:

$$\left(b^{m}\right)^{n} = b^{m \times n}$$

Examples:

$$(r^3)^2 = r^3 \cdot r^3 = r^{(3+3)} = r^6$$

$$(s^7)^4 = s^7 \cdot s^7 \cdot s^7 \cdot s^7 = s^{(7+7+7+7)} = s^{28}$$

$$(2^3)^4 = \underbrace{2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3}_{4 \text{ factors of } 2^3}$$

$$= \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{12 \text{ factors of } 2}$$

$$= 2^{12}$$

Negative Exponents

Rules:

$$x^{-n} = \frac{1}{x^n}$$

Think of the negative sign as an indicator light that says you are on the wrong floor of a two-story building.

Once you move to the correct floor, the light turns off.

Examples:

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$\frac{n^{-2}m}{7m^{-4}n^{-3}} = \frac{m^4n^3m}{7n^2}$$

$$a^3 \bullet a^{-8} = a^{-5} = \frac{1}{a^5}$$

$$3^{-2} = \frac{1}{9}$$

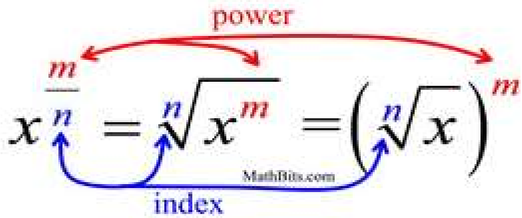
$$4^{-1} = \frac{1}{4}$$

$$7^{-5} = \frac{1}{7^5}$$

$$\frac{1}{2^{-2}} = 2^2$$

$$\frac{1}{5^{-1}} = 5^1$$

Fractional Exponents



$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

Examples:

$$\sqrt[3]{8x^3w^9z^6} = \left(8x^3w^9z^6\right)^{1/3}$$

$$= 8^{1/3}\left(x^3\right)^{1/3}\left(w^9\right)^{1/3}\left(z^6\right)^{1/3}$$

$$= 2xw^3z^2$$

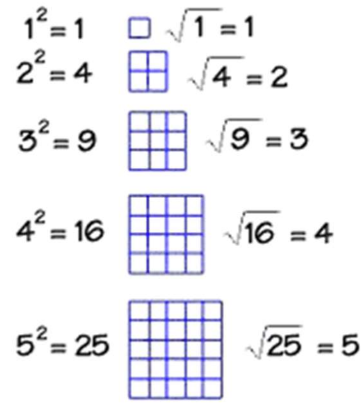
You can rewrite this guy as a 4th power. → $16^{3/4}$

$$16^{3/4} = \left(2^4\right)^{3/4} = 2^{4 \cdot 3/4} = 2^3 = 8$$

Roots and Radicals

The opposite of find a square is finding the square's **roots** or the like factors of a square.

The symbol for finding the square root is √ and is called a **radical**.



Examples:

$$\sqrt{25} = \sqrt{5 \cdot 5} = 5$$

$$\sqrt{4} = \sqrt{2 \cdot 2} = 2$$

$$\sqrt{100} = \sqrt{10 \cdot 10} = 10$$

$$\sqrt{x^2} = \sqrt{x \cdot x} = x$$

$$-\sqrt{6^2} = -\sqrt{6 \cdot 6} = -6$$

You may also see

$$\pm\sqrt{4} = \pm 2$$

which is read "plus or minus 2."